B.sc. (Computer Science) (Part 1) Examination, 2016
Paper No. 1.2
DIFFERENTIAL CALCULUS AND DIFFERENTIAL EQUATION

Time: Three Hours]   **[Maximum Marks: 50**

Note: Attempt *five* questions in all selecting *one* question from each Section. All questions carry equal marks.

Section-A

1. (a) If $y = \sin mx + \cos mx$, prove that:

$$y_n = m^n \left[1 + (-1)^n \sin 2mx\right]^{\frac{1}{2}}$$

(b) If $y = a\cos(\log x) + b\sin(\log x)$

Show that:

$$x^2 y_2 + xy_1 + y = 0$$

and $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$.

2. (a) State and prove Maclaurin's theorem.

(b) Expend $\tan^{-1} x$ in powers of $\left(x - \frac{\pi}{4}\right)$.

3. (a) If:

$$y = x_n \log x$$

prove that:

$$xy_{n+1} = n!$$

(b) Evaluate:

$$\lim_{x \to 0} \frac{\sin x - x + \frac{x^3}{6}}{x^5}$$

4. For the cardoid $r = a(1 - \cos\theta)$, prove that:

1. $\phi = \frac{\theta}{2}$
2. $p = 2a\sin^3 \frac{\theta}{2}$
3. The Pedal equation is $2ap^2 = r^3$
4. The Polar sub tangent $= 2a\sin^2 \frac{\theta}{2} \tan \frac{\theta}{2}$

Section-B

5. (a) Solve:

$$\frac{dy}{dx} = (4x + y + 1)^2$$

(b) Solve it:

$$x\frac{dy}{dx} = y - x\tan\frac{y}{x}$$

6. (a) Solve:

$$\frac{dy}{dx} - 3y\cot x = \sin 2x$$

Given $y = 2$ when $x = \frac{\pi}{2}$.

(b) Solve:

$$xdx + ydy + \frac{xdy - ydx}{x^2 + y^2} = 0$$

7. (a) Solve:

$$(D^3 + 6D^2 + 11D + 6)y = 0$$

(b) Solve:

$$(D^2 + a^2)y = \sin ax$$

8. (a) Solve:

$$(x^2 D^2 + 3xD + 1)y = \frac{1}{(1-x)^2}$$

Section-C

9. (a) Evaluate:

$$\int \frac{5x-2}{1+2x+3x^2}dx$$

(b) Evaluate:

$$\int \sqrt{2 - 3x - 4x^2}dx$$

10. (a) Evaluate $\int_a^b x^2 dx$ by summation.

(b) Evaluate $\int_0^{\frac{\pi}{2}} \log\sin x\,dx$.

......End......